Phys 4062/5062 – Lecture Nine – Classical Model for components of dipole moment

Classical Damped Harmonic Oscillator Revisited

Goals

- Understand origin of coupled equations for in phase and quadrature components of dipole moment based on classical model
- Include spontaneous emission into Bloch vector model
- find force due to absorption of photons in the Bloch model and compare to earlier expression based on classical harmonic oscillator

Begin with equation for harmonic oscillator $\ddot{x} + \Gamma\dot{x} + \omega_o{}^2 x = \frac{F(t)}{m}\cos(\omega t)$ **(233)**

- F(t) varies slowly compared to cos(ωt)
- Trial Solution: $x(t) = \mathcal{U}\cos(\omega t) - \mathcal{V}\sin(\omega t)$ **(234)**
- $\mathcal{U}$ in phase component
- $\mathcal{V}$ quadrature component
- phase lead of π/2 with respect to Fcos(ωt) if V(t) >0
  - ie $\cos(\omega t + \pi/2) = -\sin(\omega t)$

Use (234) in (233).
Equate components of cos(ωt) and sin(ωt)
Use the slowly varying envelope approximationsince envelopes of $\mathcal{U}(t)$, $\mathcal{V}(t)$, F(t) change slowly compared to ω.

$$\ddot{\mathcal{U}} \sim 0 \text{ and } \ddot{\mathcal{V}} \sim 0$$
$$\dot{\mathcal{V}} \ll \omega\mathcal{V}$$
$$\dot{\mathcal{U}} \ll \omega\mathcal{U}$$

Assume $\omega \sim \omega_0$

$$\frac{d\mathcal{U}}{dt} = (\omega - \omega_o)\mathcal{V} - \frac{\Gamma}{2}\mathcal{U}$$

$$\frac{d\mathcal{V}}{dt} = -(\omega - \omega_o)\mathcal{U} - \frac{\Gamma}{2}\mathcal{V} - \frac{F}{2m\omega} \quad \textbf{(236)}$$

Compare with (231) to see similarities with Bloch Model

Solve (236) in steady state

$$\dot{\mathcal{U}} = 0$$
$$\dot{\mathcal{V}} = 0$$

- This is a good approximation if $\mathcal{U}(t)$, $\mathcal{V}(t)$, F(t) change slowly compared 1/Γ

The steady state expressions are given by

$$\mathcal{U} = \frac{-\Delta}{\left[\Delta^2+\left(\frac{\Gamma}{2}\right)^2\right]}\left[\frac{F}{2m\omega}\right] \quad \textbf{(237A)}$$

$$\mathcal{V} = \frac{-\frac{\Gamma}{2}}{\left[\Delta^2+\left(\frac{\Gamma}{2}\right)^2\right]}\left[\frac{F}{2m\omega}\right] \quad \textbf{(237B)}$$

Compare with equations 4 and 8

Review key results from classical model

1) Dipole Moment decays at rate $\Gamma/2$ whereas energy decay sat rate Γ
2) Steady state expression for energy does not incorporate saturation
3) Absorption is related to quadrature component of dipole moment

Energy

Total energy E:

$$E = KE + PE = ½ m\dot{x}^2 + ½ m\omega_o x^2 \qquad \textbf{(238)}$$

Start with equation 234
Find dx/dt
Find new expression for energy using the slowly varying envelope approximation
Assuming $\omega \sim \omega_o$,

$$E = ½ m\omega^2(\mathcal{U}^2 + \mathcal{V}^2) \quad \textbf{(239)}$$
$$\dot{E} = m\omega^2(\mathcal{U}\dot{\mathcal{U}} + \mathcal{V}\dot{\mathcal{V}}) \quad \textbf{(240)}$$

Using (236) and 239

$$\dot{E} = -\Gamma E - \frac{F\mathcal{V}\omega}{2} \qquad \textbf{(241A)}$$

- The second term in (241A), $\frac{F\mathcal{V}\omega}{2}$ is the rate of doing work
- note that $\mathcal{V}$ has dimensions of x
- ω is the drive frequency
- The steady state solution of 241 A is

$$\boxed{E_{steadystate} = -\frac{FV\omega}{2\Gamma}} \; \textbf{(241B)} \quad \text{(predicts no saturation)}$$

Damping Rate

If F=0 in equation 233, the solution is

$$x = x_o \exp[-\Gamma t/2] \cos[\omega_o t + \varphi] \quad \textbf{(242)}$$

This shows that the dipole moment damps out with rate $\Gamma/2$

Since Energy α x^2 => E α $e^{-\Gamma t}$ **(243)**

This shows that the population damping rate is Γ

Power Absorbed

$P_{avg} = (F(t) \cos(\omega t)\, \dot{x})_{avg}$ **(244)**

Recall from (234) $\dot{x} = -\mathcal{U}\omega \sin(\omega t) - \mathcal{V}\omega\cos(\omega t)$ **(245)** (assuming $\dot{\mathcal{V}} \ll \omega\mathcal{V}$)

Using 245 in 244, it is clear that only cos(ωt) term contributes to P_{avg}

$$P_{avg} = \frac{F\mathcal{V}\omega}{2}$$ which is consistent with (241B)

So absorption is clearly related to quadrature component of dipole moment
In phase component given by (237 A) gives rise to the index of refraction