Physics 4062/5062 – Lecture Eight – Strong Field Solution

$$i\dot{c}_1 = \Omega\cos(\omega t)\, e^{i\omega_o t} c_2 \qquad \textbf{(207)}$$

$$i\dot{c}_2 = \Omega^*\cos(\omega t)\, e^{i\omega_o t} c_1$$

- Writing the cosine in exponential notation and using the rotating wave approximation, we obtain

$$i\dot{c}_1 = c_2 \frac{\Omega}{2}\exp(i(\omega - \omega_o)t) \qquad (\mathbf{213})$$
$$i\dot{c}_2 = c_1 \frac{\Omega^*}{2}\, exp(-i(\omega - \omega_o)t)$$

- combine equations in (213)

$$\frac{d^2c_2}{dt^2} + i(\omega\text{-}\omega_o)\left(\frac{dc_2}{dt}\right) + \left|\frac{\Omega}{2}\right|^2 c_2 = 0 \qquad \textbf{(214)}$$

- Use initial conditions

$$c_1(0) = 1$$
$$c_2(0) = 0$$

- To obtain:

$$|c_2(t)|^2 = \left(\frac{\Omega^2}{\Omega'^2}\right) sin^2\left(\frac{\Omega' t}{2}\right) \textbf{(215)}$$

$$\Omega'^2 = \Omega^2 + \Delta^2 \quad \textbf{(216)} \quad \text{(Generalized Rabi Frequency)}$$

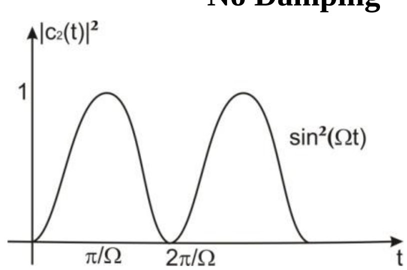
- At Resonance

$$\omega = \omega_o$$
$$\Omega' = \Omega$$

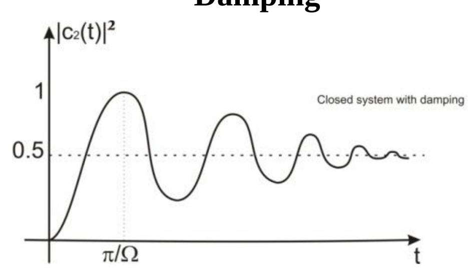
$$|c_2(t)|^2 = \sin^2\left(\frac{\Omega t}{2}\right) \qquad \textbf{(217)}$$

Population Oscillates between 2 levels at characteristic flopping frequency (Ω)

No Damping



Damping



- For $\Omega t = \pi \Rightarrow |c_2(t)|^2 = 1$ population inverted
- Behavior is different from two level system described by rate equations
- In the rate equation approach, populations of levels 1 and 2 become equal with increase in excitation rate and there is no inversion

Bloch Vector Model

Define Dipole Moment in terms of expectation value

$$p = \int \psi^*(t)(qx)\psi(t)d^3r \qquad \textbf{(218)}$$

Use $\psi(r,t) = c_1\psi_1(r)e^{-\frac{iE_1t}{\hbar}} + c_2\psi_2(r)e^{-\frac{iE_2t}{\hbar}}$ in equation 218 to obtain

$$x(t) = c_2{}^*c_1x_{21}e^{i\omega_o t}+c_1{}^*c_2x_{12}e^{-i\omega_o t} \quad \textbf{(219)}$$

Here, x_{12}=<1|$\hat{x}$|2> = $x_{12}{}^*$

$\omega_0=\omega_1-\omega_2$

$x_{11}= x_{22}=0$

Goal: Find atomic populations and components of dipole moment induced by external field

Introduce Density Matrix for ensemble of atoms

$$|\psi\rangle\langle\psi| = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \begin{pmatrix} c_1{}^* & c_2{}^* \end{pmatrix} = \begin{pmatrix} |c_1|^2 & c_1c_2{}^* \\ c_2c_1{}^* & |c_2|^2 \end{pmatrix} = \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix} \textbf{(220)}$$

ρ_{11} and ρ_{22} are the populations

ρ_{12} and ρ_{21} are coherences associated with dipole moment qx(t) where x(t) is given by equation 219

Introduce variable change

$\tilde{c_1}$ = $c_1$exp[-iΔt/2]

$\tilde{c_2}$ = $c_2$exp[-iΔt/2]

As a result, populations are unaffected, that is

$$\tilde{\rho}_{11} = \rho_{11}$$

$$\tilde{\rho}_{22} = \rho_{22}$$

The new coherences are given by

$$\tilde{\rho}_{12} = \rho_{12} e^{-i\Delta t} \qquad \textbf{(223)}$$

$$\tilde{\rho}_{21} = \rho_{21} e^{i\Delta t}$$

The atomic response becomes x(t) = $x_{12}[\widetilde{\rho_{12}}$ exp[iωt] + $\widetilde{\rho_{21}}$ exp[-iωt]] (224)

Define in phase and quadrature components of dipole moment

$$\mathcal{U} = \tilde{\rho}_{12} + \tilde{\rho}_{21}$$

$$\mathcal{V} = -i(\tilde{\rho}_{12} - \tilde{\rho}_{21}) \qquad \textbf{(225)}$$

As a result, equation 224 becomes x(t) = $x_{12}(\mathcal{U}(t)\cos(\omega t) - \mathcal{V}(t)\sin(\omega t))$

Note that $\mathcal{U}$ and $\mathcal{V}$ are components of dipole moment in frame rotating at ω

To find $\widetilde{\rho_{12}}$, $\widetilde{\rho_{21}}$ and ρ_{22}

Rewrite 213 in the new notation

$$\frac{dc_1}{dt} = c_2 \exp[i\Delta t]\frac{\Omega}{2}$$
$$\frac{dc_2}{dt} = c_1 \exp[-i\Delta t]\frac{\Omega}{2}$$

(226 A) and **(226 B)**

From 221,

$$\frac{d\tilde{c}_1}{dt} = \frac{dc_1}{dt}\exp[-i\Delta\frac{t}{2}] - i\frac{\Delta}{2}c_1\exp[-i\Delta\frac{t}{2}]$$

Multiply by i and use equations 226 A, 221 and 222
Multiply by i and use equations 226 B, 221 and 222 to obtain coupled equations

$$\frac{d\tilde{c}_1}{dt} = \frac{1}{2}[\Delta\tilde{c}_1 + \Omega\tilde{c}_2]$$
$$\frac{d\tilde{c}_2}{dt} = \frac{1}{2}[\Omega\tilde{c}_1 - \Delta\tilde{c}_2]$$

Find $\frac{d\tilde{\rho}_{12}}{dt} = \tilde{c}_1 \frac{d\tilde{c}_2 *}{dt} + \frac{d\tilde{c}_1}{dt}\tilde{c}_2 *$, and $\frac{d\tilde{\rho}_{22}}{dt}$ using the normalization condition $\rho_{11} + \rho_{22} = 1$

It can be shown that the following coupled equations are obtained

$$\dot{\mathcal{U}} = \Delta\mathcal{V}$$

$$\dot{\mathcal{V}} = \Delta\mathcal{U} + \Omega(\rho_{11} - \rho_{22}) \qquad \textbf{(229)}$$

$$\dot{\rho}_{22} = \frac{\Omega\mathcal{V}}{2}$$

Define:

$$\mathcal{W} = \rho_{11} - \rho_{22} \qquad \textbf{(230)}$$

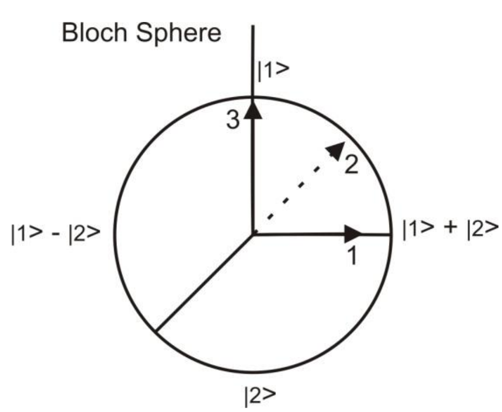
As a result the following optical Bloch equations are obtained.

$$\dot{\mathcal{U}} = \Delta\mathcal{V}$$

$$\dot{\mathcal{V}} = -\Delta\mathcal{U} + \Omega\mathcal{W} \qquad \textbf{(231)}$$

$$\dot{\mathcal{W}} = -\Omega\mathcal{V}$$

Define Bloch Vector that describes a point on the Bloch sphere of radius $|\mathcal{U}|^2+|\mathcal{V}|^2+|\mathcal{W}|^2 = 1$



$$\vec{R} = \mathcal{U}\hat{1} + V\hat{2} + W\hat{3}$$

Define interaction with field

$$\overrightarrow{\Omega'} = \Omega\hat{1} + \Delta\hat{3}$$

With these definitions, equation 231 is reduced to torque equation

$$\dot{\vec{R}} = \vec{R} \times \overrightarrow{\Omega'}$$

Model describes precession of Bloch vector around Omega Prime (vector symbol).

Time dependent Components of Bloch vector give populations and coherences