

Phys 4062/5062 – Lecture Five – Heating due to Spontaneous Emission

Outline

1. Recall Cooling Rate
2. Calculate Heating Rate due to Fluctuations
3. Use Photon Absorption Rate – R
4. Find Equilibrium Temperature – Doppler Limit T_{Doppler}
5. Explore Estimates for Temperature Limits

Heating

- prediction of $E = E_0 \exp[-t/\tau]$, that energy $\rightarrow 0$ is unphysical
- so far: we have ignored heating due to fluctuations in force
- both absorption and spontaneous emission involve momentum transfer in unit of $\hbar k$

Atom executes random walk with step size $\hbar k$ due to these processes

- number of steps is proportional to number of photons absorbed/emitted
- absorption equally probable from both beams for $v \sim 0$ atoms
- absorption + spontaneous emission – 2 steps in random walk one from random absorption and one from random emission

$$\text{Number of Steps, } dN = 2 R_{\text{total}} dt \quad (21)$$

$$R_{\text{total}} = R_+ + R_- \quad (\text{Total Absorption Rate}) \text{ where } R \text{ is given by (13)}$$

ID Random Walk

$$\text{Average momentum, } \langle p \rangle = 0 \quad (22)$$

Mean Square

$$\langle p^2 \rangle = N(\hbar k)^2 \quad (23)$$

Rate of Heating

$$\frac{dE}{dt}_{\text{heat}} = \left(\frac{1}{2M} \right) \left(\frac{d\langle p^2 \rangle}{dt} \right) \quad (24)$$

Using equations (23), (21), (13), and (24),

$$\frac{dE}{dt}_{\text{heat}} = \left[\left(\frac{(\hbar k)^2}{2M} \right) \left(\frac{4 \left(\frac{\Gamma}{16} \right) \Gamma}{1 + \frac{4\Delta^2}{\Gamma^2}} \right) \right] \quad (25)$$

To find equilibrium temperature, assume steady state conditions

$$\frac{dE}{dt} = \frac{dE}{dt}_{\text{cool}} + \frac{dE}{dt}_{\text{heat}} = 0$$

Using equations (18A), (17) and (25)

$$E_{\text{steadystate}} = \left[\frac{1 + \frac{4\Delta^2}{\Gamma^2}}{2 \left(\frac{\Delta}{\Gamma} \right)} \right] \left[\frac{\hbar \Gamma}{8} \right] = \frac{k_B T}{2} \quad (26)$$

- energy per degree of freedom = $\frac{k_B T}{2}$ according to equipartition theory

Find expression for temperature T

Minimize expression for temperature to find $T_{\min} = \hbar\Gamma/2k_B$ at $\Delta = -\Gamma/2$

$$T_{\min} = T_{\text{Doppler}} \text{ (Doppler Limit)}$$

- estimate T_{Doppler} for Rb
- estimate Doppler FWHM of cold RB

Note: Temperature of molasses of MOT is typically well below T_{Doppler} because of the mechanism of polarization gradient cooling

Explore Estimates for temperature limits

1. Recoil limit T_γ
2. Spectral Limit T^*
3. Doppler Limit T_D

Notice $T_D = (T^*T_\gamma)^{1/2}$