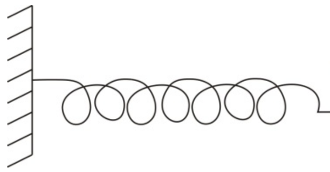
Physics 4062/5062 – Lecture Three –Force due to Absorption of Light

Atom excited by near resonant light can be modeled as a damped, driven harmonic oscillator



ω_o is the resonant frequency of the spring
Γ is the damping rate in (units s^{-1})
Drive: $E = E_o\cos(\omega t)$ **(1)**
ω is the angular frequency of the drive (laser)
E_o is the amplitude

Displacement x(t) is given by the solution to:

$$\ddot{x} + \Gamma\dot{x} + \omega_o^2 x = \frac{qE_o\cos\omega t}{m} \quad \textbf{(2)}$$

- Trial solution: $x(t) = x_o\cos(\omega t - \varphi)$ **(3)**

Define Detuning $\Delta = \omega - \omega_o$

Assume $\Delta, \Gamma << \omega_o$

Find

$$x_o = \frac{\frac{qE_o}{m}}{2\omega_o\left(\Delta^2 + \frac{\Gamma^2}{4}\right)^{\frac{1}{2}}} \quad \textbf{(4)}$$

$$\varphi = \tan^{-1}\left(-\frac{\Gamma}{2\Delta}\right) \quad \textbf{(5)}$$

Since electron is damped, power absorbed during each cycle of optical field is given by

$$P_{inst} = Fv_e = qE\dot{x} \quad \textbf{(6)}$$

- F is the force on electron
- v_e is the velocity of electron

$$P_{inst} = F\dot{x} = -qE_o\omega x_o[\cos(\omega t)\sin(\omega t)\cos(\varphi) - \cos^2(\omega t)\sin(\varphi)]$$

$$P_{avg} = \langle P \rangle = \frac{1}{T}\int_0^T P_{Inst}\, dt$$

Where $T = \frac{2\pi}{\omega}$ is the period

$$\boxed{\langle P\rangle = \frac{1}{2} q E_o \omega x_o \sin\phi} \quad \textbf{(7)}$$

Using **(4)** and **(5),** the identity $\sin\phi = \frac{\tan\phi}{\sqrt{1+\tan^2\phi}}$ and assuming $\omega \sim \omega_o$

$$\langle P\rangle = \frac{q^2}{2m}\left[\frac{\Gamma}{4\Delta^2+\Gamma^2}\right] {E_o}^2 \quad \textbf{(8)}$$

- Define Rate of Photon Absorption R

$$R = \frac{<P>}{PhotonEnergy}$$

Use the definition of intensity, $I = \frac{1}{2}\varepsilon_o c {E_o}^2$ and the saturation intensity $I_s = \frac{\varepsilon_o m c \Gamma^2 \hbar\omega}{q^2}$ to show that

$$R = \frac{I/I_s}{1+4\frac{\Delta^2}{\Gamma^2}}\Gamma \qquad \textbf{(13)}$$

Force due to absorption is given by

$F_{abs} = R\hbar k$ **(9)** which gives

$$F_{abs} = \frac{I/I_s}{1+4\frac{\Delta^2}{\Gamma^2}}\hbar k\Gamma \qquad \textbf{(14)}$$

- $F \propto I$ but no saturation
- For $I = I_s$, $F_{max} = \hbar k\Gamma$ (low intensity result)
- Model is good only for $I < I_s$, where $I_s \sim 1$ mW/ cm^2

Note: correct high intensity expression includes the effect of stimulated emission so that

$F_{max} = \hbar k\Gamma/2$

As an exercise, graph F versus Δ and make observations about what this means.

As an exercise, estimate the maximum acceleration for Rb.