**Phys 4062/5062 – Lecture Ten**

**Prediction for Force due to Absorption of Light– Bloch Equation Model**

Energy of two level atom:

$$E = \rho_{22}\hbar\omega_o \qquad \textbf{(247A)}$$

$\rho_{22}$ fractional excited state population
Compare coupled equations for $\mathcal{U}$ and $\mathcal{V}$ using the classical model (236) with 3rd optical Bloch equation (229) and introduce damping into (229)

$$\dot{\rho}_{22} = \frac{\Omega\mathcal{V}}{2} - \Gamma\rho_{22} \qquad \textbf{(247B)}$$

Based on equation 247A, the above equation is similar to equation 241A, which describes energy damping in a harmonic oscillator

Recall $\dot{E} = -\Gamma E - \frac{F\mathcal{V}\omega}{2}$ **(241A)**

If $\Omega = 0$ (no drive/excitation), then the solution of 247B is given by

$$\rho_{22} = \rho_{22}(t=0)\exp[-\Gamma t]$$

So the rate of population damping is $\Gamma$ in the optical Bloch equation model as well.

Since $\mathcal{W} = \rho_{11} - \rho_{22}$, the 3rd equation in (229) becomes 3rd equation n (231)

$$\dot{\mathcal{W}} = -\Omega\mathcal{V} - \Gamma(\mathcal{W} - 1) \qquad \textbf{(248A)}$$

We now include damping terms in the first two optical Bloch equations (OBE) in 231. To do this correctly, we recall that the damping rate of the dipole moment should be $\Gamma/2$ as noted in the previous lecture and is evident from equation 236

| Classical with Damping (236) | OBE Without Damping (231) | OBE with Damping (248A, B, C) |
|---|---|---|
| $\dot{\mathcal{U}} = \Delta\mathcal{V} - \frac{\Gamma}{2}\mathcal{U}$ | $\dot{\mathcal{U}} = \Delta\mathcal{V}$ | $\dot{\mathcal{U}} = \Delta\mathcal{V} - \frac{\Gamma}{2}\mathcal{U}$ |
| $\dot{\mathcal{V}} = -\Delta\mathcal{U} - \frac{\Gamma}{2}\mathcal{V} - \frac{F}{2m\omega}$ | $\dot{\mathcal{V}} = -\Delta\mathcal{U} + \Omega\mathcal{W}$ | $\dot{\mathcal{V}} = -\Delta\mathcal{U} + \Omega\mathcal{W} - \frac{\Gamma}{2}\mathcal{V}$ |
| | $\mathcal{W} = -\Omega\mathcal{V}$ | $\mathcal{W} = -\Omega\mathcal{V} - \Gamma(\mathcal{W} - 1)$ |

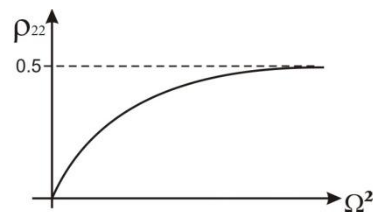
We now find the steady state solutions of 248 just like we found the steady state solutions of 236 to get 237 A and 237 B.

Solve (248) with $\dot{\mathcal{V}} = 0, \dot{\mathcal{U}} = 0,$ and $\dot{\mathcal{W}} = 0$

These solutions are valid if $t \gg \frac{1}{\Gamma}$ that is $t \gg \tau_N$

In steady state, we obtain

$$\begin{pmatrix} \mathcal{U} \\ \mathcal{V} \\ \mathcal{W} \end{pmatrix} = \frac{1}{\Delta^2 + \frac{\Omega^2}{2} + \frac{\Gamma^2}{4}} \begin{pmatrix} \Omega\Delta \\ \Omega\frac{\Gamma}{2} \\ \Delta^2 + \frac{\Gamma^2}{4} \end{pmatrix} \quad \textbf{(249)}$$




From equation 249

for $\Omega \rightarrow \infty$,

$$\rho_{22} = \frac{1-\mathcal{W}}{2} = \frac{\frac{\Omega^2}{4}}{\Delta^2 + \frac{\Omega^2}{2} + \frac{\Gamma^2}{4}} \qquad \textbf{(250)}$$

Note: $\Omega^2 \propto I$
(250) predicts $\rho_{22} \rightarrow$ ½ for high intensity (populations equalized).

Thus the OBE predict saturation. The classical model did not predict saturation.

**Force due to Absorption of Photons – Revisited using OBEs**

Recall Radiation Pressure

$$P_{Rad} = \frac{F_{Rad}}{A} = \frac{I}{c}$$

$$(P_{Rad})_{Avg} = \tfrac{1}{2}\, \varepsilon_o E_o^2 = \frac{I_{avg}}{c}$$

In earlier treatment in the course, we used $F_{(\text{Abs force or scattering force})} = \hbar k R$ **(9)**

R is the rate of photon absorption.

From previous lecture

$$R = \Gamma \rho_{22} \qquad \textbf{(251)}$$

Using (250)

$$R = \left(\frac{\Gamma}{2}\right)\left[\frac{\frac{\Omega^2}{2}}{\Delta^2 + \frac{\Omega^2}{2} + \frac{\Gamma^2}{4}}\right] \quad \textbf{(252)}$$

compare with equation 13

Define:

$$\sqrt{\frac{I}{Is}} = \left(\frac{\sqrt{2}\Omega}{\Gamma}\right) \quad \textbf{(253)}$$ compare with equation 12

Using equation (9),

$$F_{abs/scat} = \hbar k \left(\frac{\Gamma}{2}\right)\left[\frac{\frac{I}{I_S}}{1+\left(\frac{I}{I_S}\right)+4\left(\frac{\Delta}{\Gamma}\right)^2}\right] \quad \textbf{(254)}$$ compare with equation 14

- as I → ∞, $F = \hbar k\Gamma/2 = F_{max}$
- (254) is correct high intensity expression for force.

Therefore it is clear that OBEs predict correct intensity dependence of force.

Both the OBE model and the classical model predict the correct frequency dependence of the force.