Phys 4061- Lecture Nine

Outline

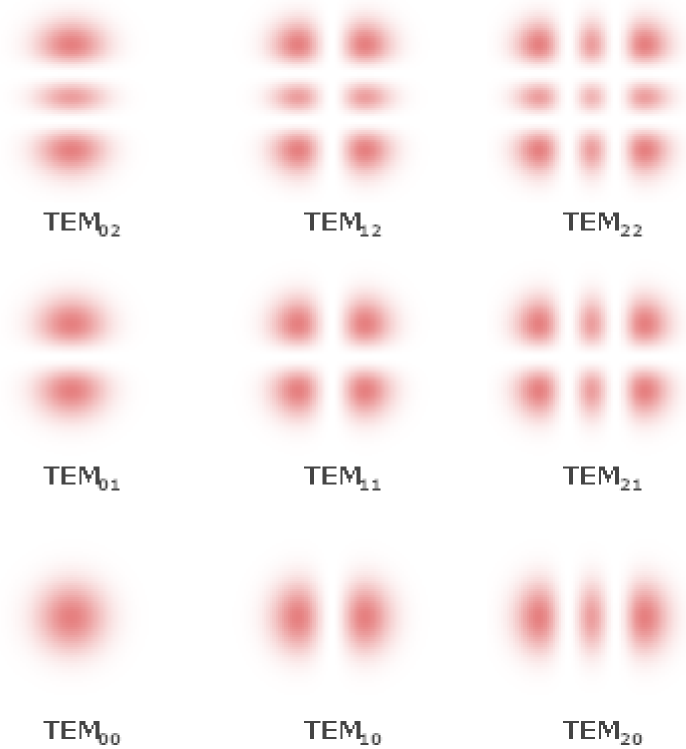
Hermite-Gaussian

$TEM_{\ell m}$

ℓ intensity zeros
$\ell + 1$ maxima

m intensity zeros
m+1 maxima

for $\ell \neq 0$/ $m \neq 0$

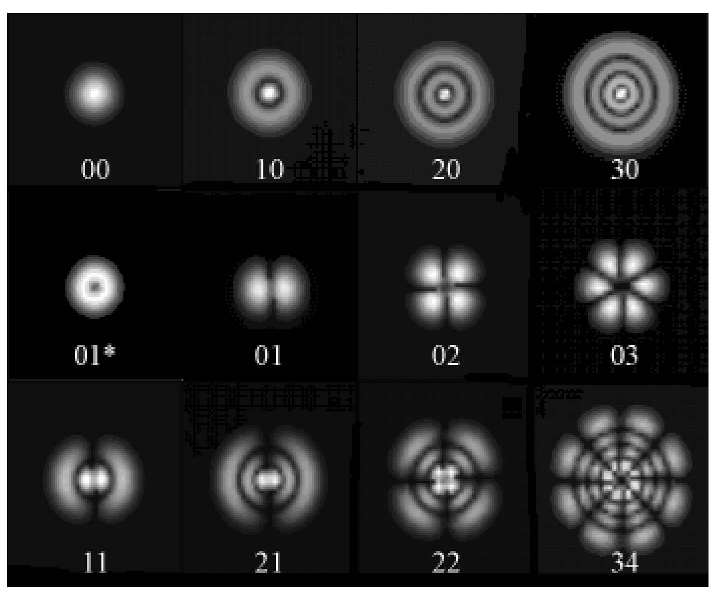


Mode frequencies depend on cavity geometry

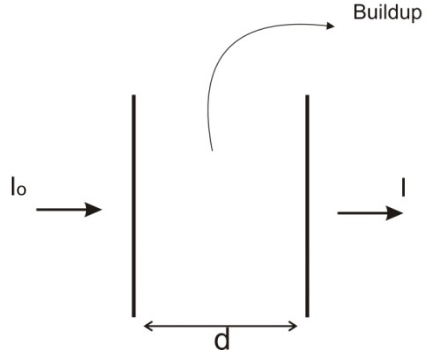
Higher order modes (curvature of mirror)

Laguerre- Gaussian

Are cylindrically symmetric



Spatial Modes



Longitudinal Cavity Modes

$\Delta\nu \equiv c / 2d$

- Quantum states of an EM field
- Photons occupying cavity mode states is analogous to electron occupying atomic states

  $\Delta\omega\Delta t \sim 1$ (classical optics)

  $\Delta(\hbar w)\Delta t \sim \hbar$ (quantum optics) – mode energy is uncertain because of uncertainty in time which photon leaves cavity

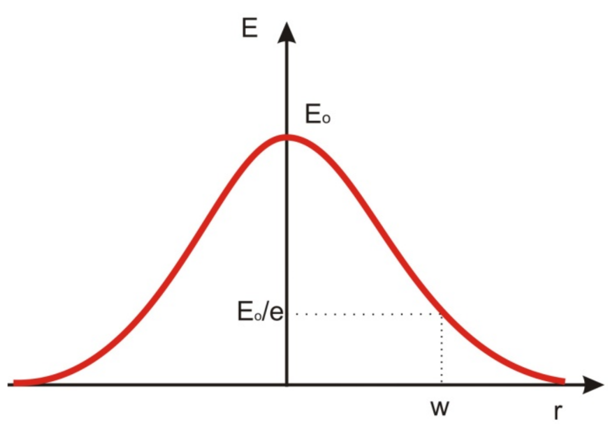
Gaussian Beams

- Cavity Modes represent longitudinal modes that correspond to standing waves along axis
- Frequencies depend on separation on mirror separation d
- Light distribution in transverse direction or perpendicular to cavity axis are represented by transverse modes

Properties:

This is a fundamental mode of laser cavity that represents a particular transverse mode
Natural confinement, ie: transverse confinement without mirrors – solution to Maxwell's equations
Gaussian spatial profile at any location
Smallest possible angular spread for a given initial beam diameter
Spread due to diffraction only
No oscillations in transverse profile

Spatial Profile is a smooth function



$$E(r,z) = E_o(w_o/w(z))\exp[-r^2/w(z)^2]$$
$$r^2 = x^2 + y^2 \quad \text{(radial coordinates)}$$

z = propagation direction
$w(z)$ = spot size or radius
w_o = minimum spot size at $z=0$

- The curvature of wave front changes along z
- At a small distance from z axis the wave front can be approximated as being spherical

$$w^2(z) = w_o^2[1+(z/z_o)^2] \quad (2)$$
$$R(z) = z[1+(z_o/z)^2] \quad (3)$$

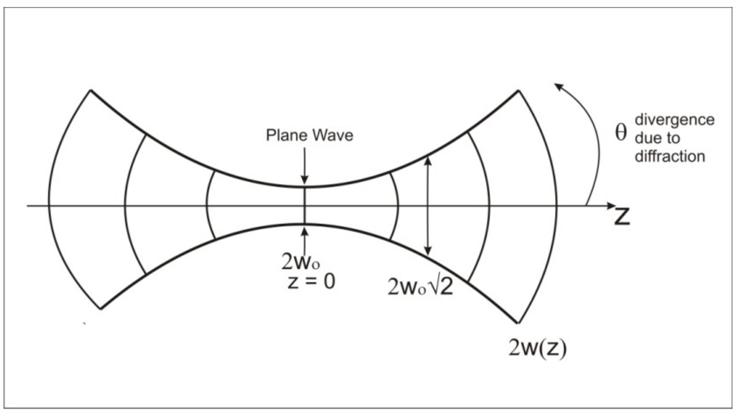
(2) and (3) obtain by solving Maxwell's equations for cavity

$$z_o = \pi w_o^2/\lambda \quad \text{(Rayleigh Range)}$$

Two Parameters Specify Gaussian Beam

- w_o and $w(z)$ for a given λ

Note that radius of curvature changes sign as beam propagates through focal plane (z=0)
Notice that the wavefronts are plane waves at z-0



Rayleigh Range z_o

$$w(z = z_o) = \sqrt{2}w_o$$
$$A(z_o) = 2A(z=0)$$

Here A is the area at Rayleigh range which is twice the area at z= 0 since area is proportional to w^2

- $2z_o$ = cofocal beam parameter
- For $z >> z_o$, $w(z) \sim w_o(z/z_o)$

Divergence Half Angle

From Geometry, $\theta = w(z)/z = w_o/z_o$
Using the definition of z_o in above equation

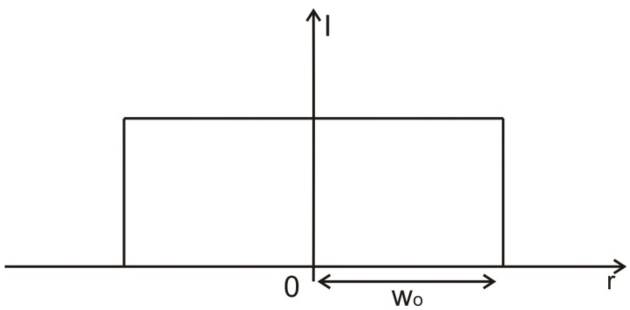
$$\theta \sim \lambda/\pi w_o$$

Recall diffraction through circular aperture

$$\theta_{full} \sim \lambda/D$$

Where D is the beam diameter

Practical Problem

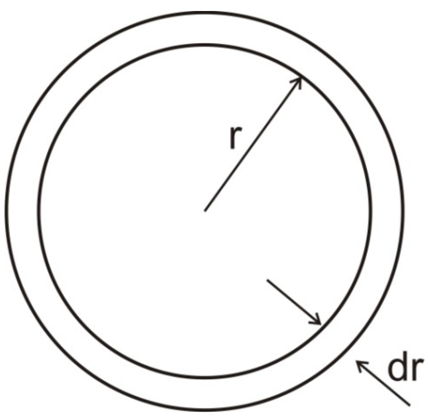


An important practical problem is to infer the peak intensity from a measurement of the beam power
Assume circularly polarized symmetric beam with uniform intensity
Spatial profile is top hat so

$$I_{max} = P/\pi w_o^2$$

Peak Intensity for Gaussian Beam

Divide the beam into circular annuli. Consider annulus of radius r and thickness dr.



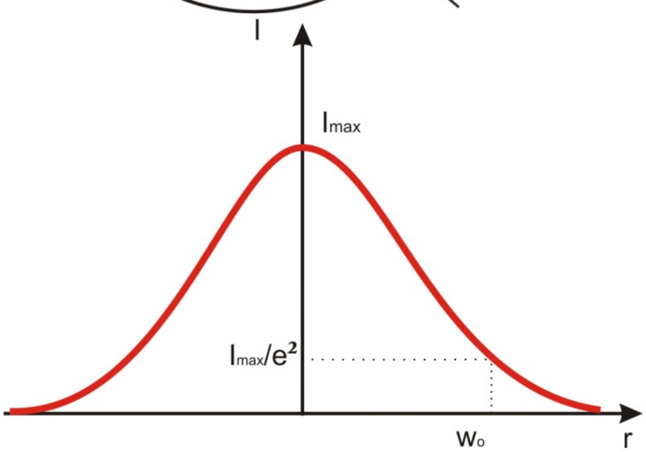
$dA = 2\pi r\, dr$

$P = \int I(r,z) dA = \int_0^\infty I(r,z) 2\pi r\, dr$

Here $I(r,z) = \frac{1}{2} c\, \varepsilon_o E^2(r,z)$

$E(r,z) = E_o \frac{w_o}{w(z)} e^{-r^2/w^2(z)}$ [Gaussian Profile]

Where $E_o = E(r = 0, z = 0)$



Using the identity $\int_0^\infty e^{-\beta u^2} du = \frac{1}{2}\sqrt{\frac{\pi}{\beta}}$,

show that $P = \frac{\pi}{2} w_o{}^2 \frac{1}{2} c \varepsilon_o E_o{}^2 = \frac{\pi}{2} w_o{}^2 I_{max}$

So that $I_{max} = 2P/\pi w_o{}^2$

Similarly using $E(r,z) = E_o \frac{w_o}{w(z)} e^{-r^2/w^2}$

show that $I(r = 0, z) = \frac{P}{\left(\frac{1}{2}\right)\pi w(z)^2}$