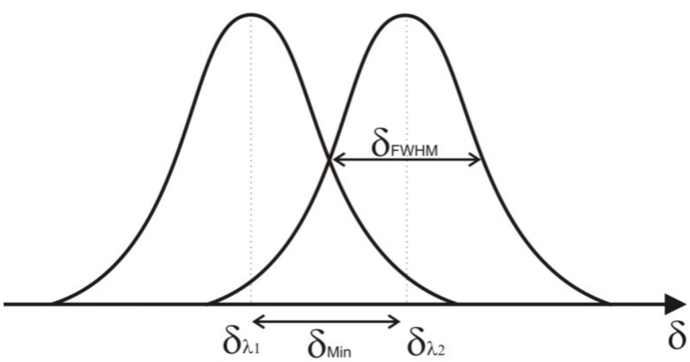
Phys 4061 – Lecture Eight – Resolution or FPI

So far we assumed a single incident wavelength λ. Now we distinct wavelengths λ_1 λ_2 separated by Δλ. ie $\lambda_2 = \lambda_1 + \Delta\lambda$.

What is the smallest Δλ that can be resolved?
The resolution of the FPI represents the spectral width of the source due to instrumental broadening.

Condition for Resolving λ_1 and λ_2



Rayleigh Criterion
$\delta_{min} = \delta_{FWHM}$ [minimum separation bet peaks that can be resolved]

$\delta_\lambda = \left(\frac{2\pi}{\lambda}\right)p = \left(\frac{2\pi}{\lambda}\right)2d\cos\theta$

For normal incidence

$$\delta_{\lambda 1} = \left(\frac{2\pi}{\lambda}\right)2d = \frac{4d\pi}{\lambda} = 2m\pi$$

$$\delta_{\lambda 2} = \frac{4d\pi}{\lambda + d\,\lambda}$$

Note:

$$\delta_{min} = \delta_{FWHM} = \frac{4}{\sqrt{F}} = \frac{2\pi}{\mathcal{F}}$$

$$\delta_{min} = \delta_{\lambda 1} - \delta_{\lambda 2}$$

$$\delta_{min} = \delta_{FWHM} \rightarrow \frac{2\pi}{\mathcal{F}} = \delta_{\lambda 1} - \delta_{\lambda 2}$$

$$2\pi/\mathcal{F} = \frac{4\pi d\,(d\lambda)}{\lambda(\lambda + d\,\lambda)} \quad (1)$$

Define Resolving Power

$$\mathcal{R} = \frac{\lambda}{d\lambda}$$

Solve for $\frac{\lambda}{d\lambda}$ from equation 1 so that $\frac{\lambda}{d\lambda} \sim \frac{2\mathcal{F}d}{\lambda} = \mathcal{F}m$

Example:

$\mathcal{F} \sim 100$
$\lambda \sim 1\ \mu m$
$d = 100$ mm

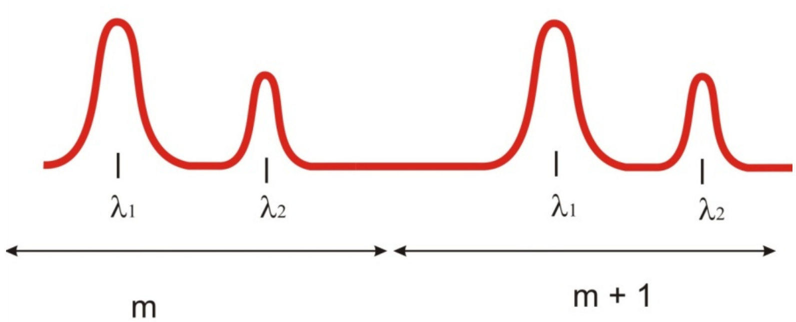
Estimate:

$$m = 2d/\lambda = 2 \times 10^5$$
$$d\lambda = \lambda/\mathcal{F}m = 5 \times 10^{-14}\ m$$
$$\Delta\nu = (c/\lambda^2)d\lambda = 15\ MHz$$

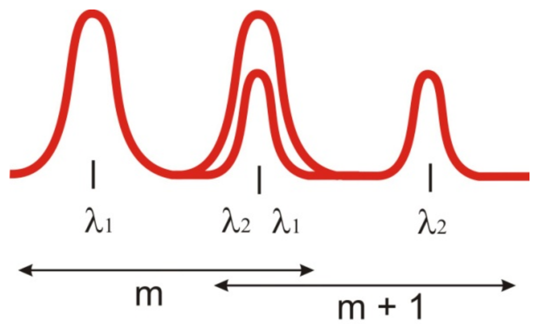
The same estimate can be obtained by calculating the number of round trip in the cavity.

Free Spectral Range represents the working range with no overlapping orders

Non Overlapping Case



Overlapping Case



Limiting Case

$$(m+1)\lambda_1 = m\lambda_2$$

- Remember that $\lambda_2 = \lambda_1 + \Delta\lambda$
- $\Delta\lambda$ is the free spectral range (FSR) without overlap

$$(m+1)\lambda_1 = 2d \quad (1)$$

$$m(\lambda_1 + \Delta\lambda) = 2d \rightarrow m = \frac{2d}{\lambda_1+\Delta\lambda} \quad (2)$$

use (2) in (1)

$$\left(\frac{2d}{\lambda_1+\Delta\lambda} + 1\right)\lambda_1 = 2d$$

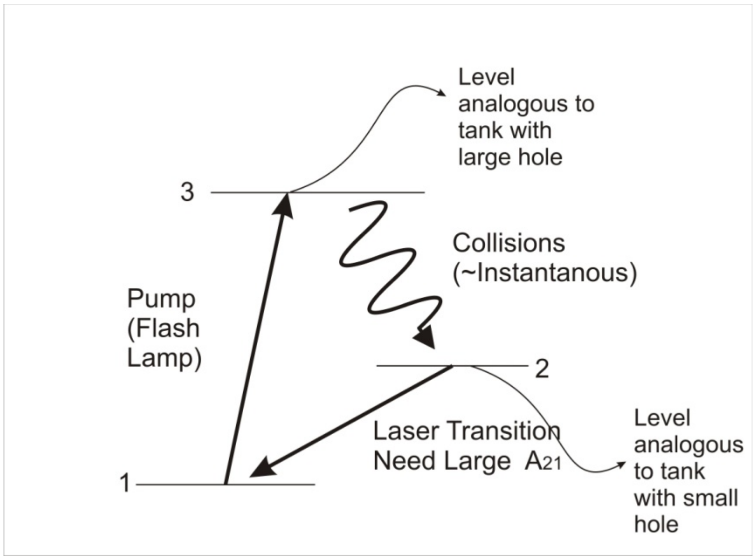
Solve for $\Delta\lambda$

$$\Delta\lambda = \Delta\lambda_{FSR} = \frac{\lambda_1^2}{2d}$$

$$\Delta\nu_{FSR} = \frac{c}{\lambda_1^2}\Delta\lambda_{FSR} = \frac{c}{2d}$$

Recall that this is spacing between adjacent modes

Optically Pumped Gas Laser



- Population inversion between 2 and 1
- No population in 3 because of quick relaxation

Expected Requirement: Pump faster to 2 than decay by spontaneous emission from 2-1

$$dn_2/dt = (R_{21} + A_{21})n_2 – (R_p + R_{12})n_1$$

$$n_3 = 0 \Rightarrow dn_1/dt = - dn_2/dt$$

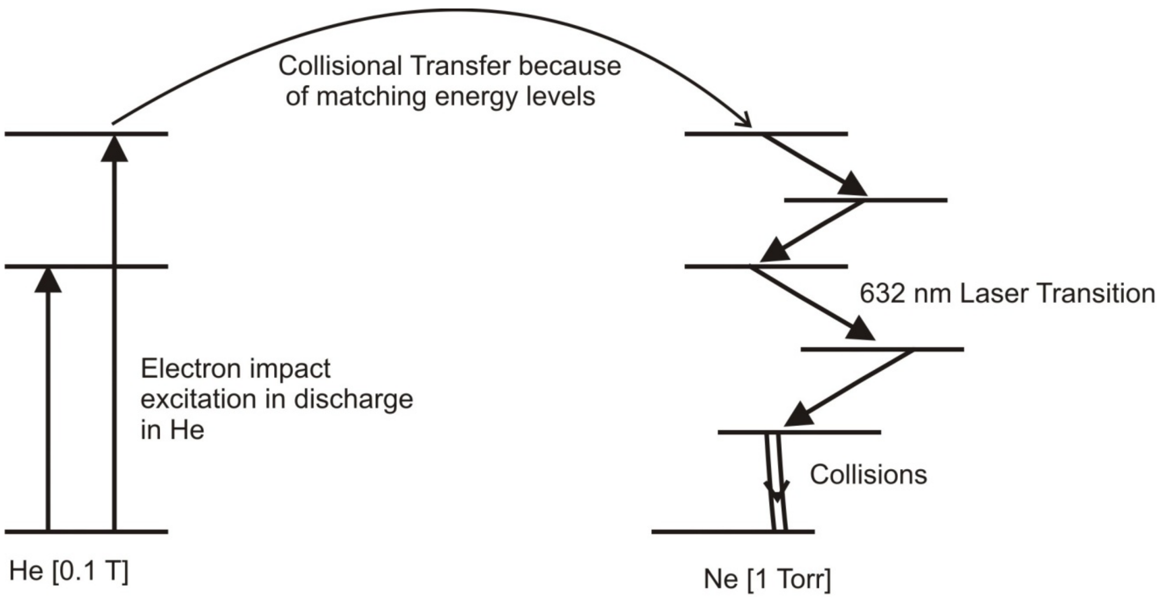
Where $R_{21} = B_{21}\rho_\nu$ (Stimulated Rate), $R_{12} = B_{12}\,\rho_\nu$ (Absorption Rate), and R_p = pump rate

- use $R_{12} = R_{21}$ and steady state condition $dn_2/dt = 0$
- Define: $\Delta n = n_2 – n_1$

Find $\Delta n = R_p – A_{21}/R_p + A_{21} + 2R_{12}$
As expected, the condition for maintaining an inversion is consistent with the expected requirement that $R_p > A_{21}$.
Note: Δn should also satifiy previously derived condition

$$\Delta n \geq \frac{8\pi\gamma_{th}\pi\Delta\nu}{2A_{21}\,\lambda^2}$$

He-Ne Laser



He-Ne mixture is gain medium inside 30 cm cavity with $\mathcal{F}$ ~ 30