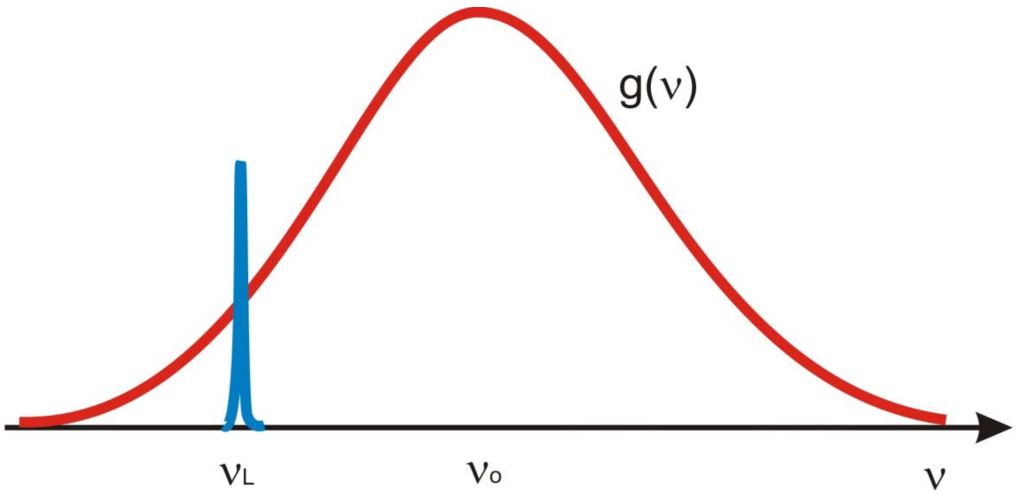
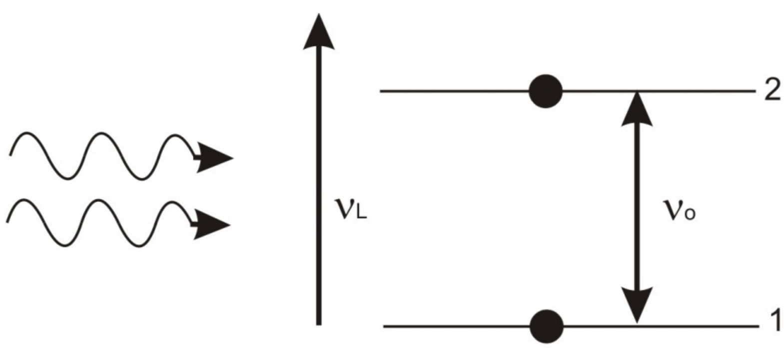
4061- Lecture Five – Gain Coefficient for a Laser

2 Level Atoms Stimulated by Monochromatic Source



- ν_L is the frequency of the monochromatic source

2 Level Atoms Exposed to Monochromatic Source



Rate for Stimulated Emission

$$R_{21} = B_{21}\rho_\nu$$

Rate for Absorption

$$R_{12} = B_{12}\rho_\nu$$

Photons Added to the Beam/ Unit Time = n_2R_{21}
Photons Removed from the Beam/ Unit Time = n_1R_{12}
Change in photons in Beam/ Unit Time = $n_2R_{21} - n_1R_{12} = \Delta n\, R$

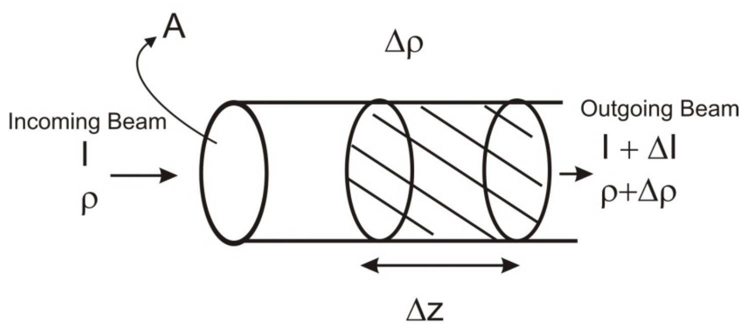
- Δn is the difference in population density
  Note: $R= R_{21}= R_{12}$

Note: We can neglect spontaneous emission as only as small fraction of events will produce photons along the same path as the beam.

Rate of Change of Energy Density

$$\Delta\rho/\Delta t = \Delta n R h\nu$$

Here $h\nu$ is the energy of the photon



- Δz is the thickness of the gain medium
- The incident beam will transit through the gain medium in time Δt

$$\Delta t = \Delta z/c$$

Increase in energy density

$$\Delta\rho = \Delta n R h \frac{\Delta z}{c}$$

Since $\Delta\rho = \Delta I/c$ and since $R = B = A_{21}c^3/8\pi h\nu^3$

$$\Delta I = A_{21}\,(c^2/8\pi\nu^2)\, g(\nu)\, I\, \Delta n\, \Delta z$$

$$\lim_{z\to 0} \Delta I/\, \Delta z \rightarrow dI/dz = \frac{\lambda^2}{8\pi} A_{21}\, g(\nu)\, \Delta n\, I = \gamma(\nu)\, I$$

Here $\gamma(\nu)$ is the gain coefficient which is the fractional change in intensity per unit propagation distance in units of m^{-1}

$$\gamma(\nu) = \frac{\lambda^2}{8\pi} A_{21}\, g(\nu)\, \Delta n = (1/\Delta z)(dI/I)$$

Interpretation

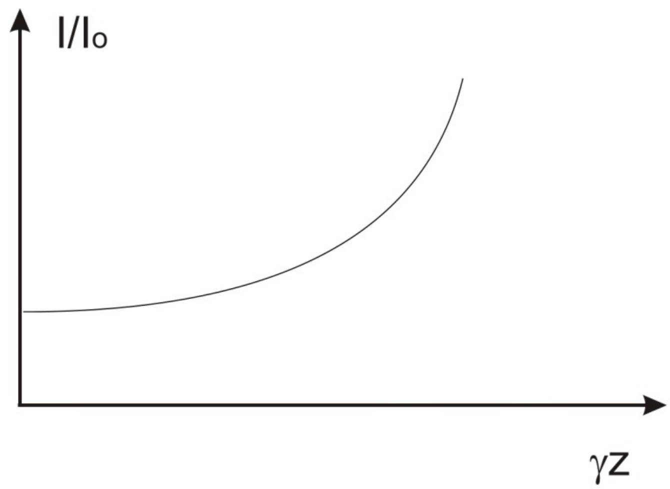
γ has to be positive for amplification $\rightarrow$ $n_2 > n_1$
Need a population inversion and this requires a non-equilibrium distribution
Amplification (Lasers) require a pump to create non-equilibrium distribution
Desirable to work at peak of gain curve
Need large A_{21} for increasing $\gamma(\nu)$
Shorter wavelength results in smaller γ

Assume γ is positive and independent of z,

$$\frac{dI}{I} = \gamma(\nu)dz$$

$$\int_{I_o}^{I} \frac{dI}{I} = \int_0^z \gamma(\nu)dz$$

$$I = I_o e^{\gamma(\nu)z}$$



- Indefinite increase is unphysical
- In practice Δn decreases and limits increase
- A pump is needed to counteract decrease in Δn

Define Gain Cross Section

$$\gamma(\nu) = \frac{\lambda^2}{8\pi} A_{21}\, g(\nu)\, \Delta n = \sigma(\nu)\, \Delta n$$

- σ(ν) is the gain cross section
- contains single atom properties
- is the effective absorbing area of atom (can be larger than area of atom)
- units of m^2

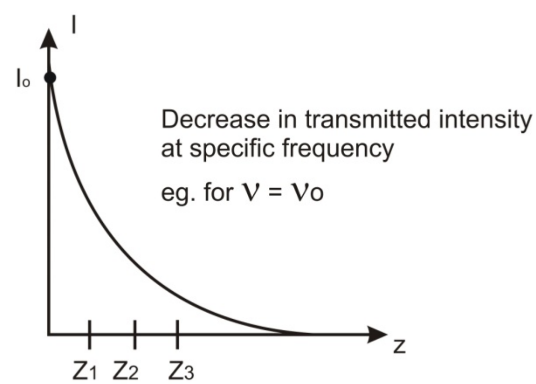
Attenuation

Assume $n_2 << n_1$

- note that γ(ν) is negative
- with $n_2 = 0$, define

$$\alpha(\nu) = n_1 \sigma_{abs}(\nu)$$

- $\sigma_{abs}(\nu)$ is the absorption coefficient



Beers Law

$$I(z) = I_o e^{-\alpha z}$$

- αz is the optical depth